Migdal Effect in DM direct detection experiments

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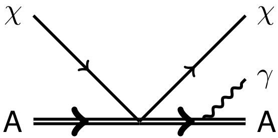
Dark Matter

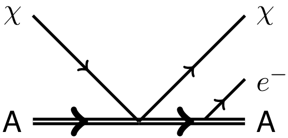
- The existence of DM is overwhelmingly supported by numerous cosmological and astrophysical observations on a wide range of scales
- DM can be directly detected by searching for it's scattering with atomic nuclei
- The Migdal effect enables to detect signals even for sub-GeV DM
- WIMP's are the most extensively studied DM candidates in the universe and couple to the SM via interactions similiar in strength to the weak interaction
- The WIMP's circular speed around the sun is estimated to be $v \approx 220km/s$, so the WIMP's recoil the atomic nuclei with a typical momentum transfer of $\mathcal{O}(100MeV)$ for the target nucleus with mass $m_N \approx 100GeV$

Ionization/excitation

It is usually assumed that the the atomic electrons of the target atom immediately follow the motion of the recoiled nucleus.
In reality it takes some time for the electrons to ”catch up”, which can result in ionization, scintillation and excitation processes.



(a)excitation



(b)ionization

Migdal effect

Migdal effect

The Migdal effect provides a theoretical framework to describe ionization/excitation processes of bound atomic electrons, induced by the recoil of an atomic nucleus. Also ionization/excitation of inner orbitals are described by Migdal's approach.

- The Migdal effect approximates the state of the electron cloud after the nuclear recoil by

$$|\Phi_{ec}^F\rangle = e^{-im_e \sum_i \mathbf{v}\cdot\mathbf{x}_i} |\Phi_{ec}^I\rangle \,. \quad (1)$$

- The probability of ionization/excitation is

$$\mathcal{P} = |\langle\Phi_{ec}^F|\Phi_{ec}^*\rangle|^2, \quad (2)$$

where $|\Phi_{ec}^*\rangle$ denotes an ionized/excited state.

- In the following analysis the final state ionization/excitation is treated seperately from the nuclear recoil.

Nuclear recoil

Hamiltonian of the atomic system

$$H_A \simeq \frac{\mathbf{p}_N^2}{2m_N} + H_{ec}(\mathbf{x}_N), \tag{3}$$

$$H_{ec}(\mathbf{x}_N) = \sum_{i=1}^{N_e} \frac{\mathbf{p}_i^2}{2m_e} + V(\mathbf{x}_i, \mathbf{x}_j, \mathbf{x}_N), \tag{4}$$

satisfying the **energy eigenequation**:

$$H_A \psi_E(\{\mathbf{x}\}, \mathbf{x}_N) = E_A \psi_E(\{\mathbf{x}\}, \mathbf{x}_N) \tag{5}$$

- For the treatment of nuclear recoil the atomic system is well characterized by the non relativistic Hamiltonian since the recoil energy is of $\mathcal{O}(100keV)$.

Atomic system at rest

Eigenstates of the electron cloud

The eigenstates of $H_{ec}(\mathbf{x}_N)$ are determined by:

$$H_{ec}(\mathbf{x}_N)\phi(\{\mathbf{x}\},\mathbf{x}_N) = E_{ec}\phi(\{\mathbf{x}\},\mathbf{x}_N). \tag{6}$$

Born Oppenheimer approximation

The dominant contribution of the total Hamiltonian H_A comes from the electron cloud H_{ec}, such that the energy eigenfunction of the whole **atomic system at rest** can be approximated by

$$\psi_{E_A}^{(rest)}(\{\mathbf{x}\},\mathbf{x}_N) \equiv \phi(\{\mathbf{x}\},\mathbf{x}_N), \tag{7}$$

$$H_A\psi_{E_A}^{(rest)}(\{\mathbf{x}\},\mathbf{x}_N) \simeq E_A\psi_{E_A}^{(rest)}(\{\mathbf{x}\},\mathbf{x}_N), \tag{8}$$

which is nothing but the **Born-Oppenheimer approximation.**

Energy eigenstates of a moving atom

- The energy **eigenstates of a moving atom** can be immediately obtained by a **Galilei transformation**

$$\psi_{E_A}(\{\mathbf{x}\}, \mathbf{x}_N) \simeq U(\mathbf{v})\psi_{E_A}^{\text{(rest)}}(\{\mathbf{x}\}, \mathbf{x}_N) \tag{9}$$

- The atomic wave function gets boosted in the direction of the nuclear recoil, such that the eigenstate of the atomic wave function is approximated by

$$\psi_{E_A}(\{\mathbf{x}\}, \mathbf{x}_N) \simeq \underbrace{e^{i\mathbf{p}_N \cdot \mathbf{x}_N} e^{i\sum_{i=1}^{N_e} \mathbf{q}_e \cdot \mathbf{x}_i}}_{U(\mathbf{v})} \psi_{E_A}^{\text{(rest)}}(\{\mathbf{x}\}, \mathbf{x}_N) \tag{10}$$

Hartree Fock method

- The HF-theory is a **selfconsistent theory** using the concept of **partial waves**, where every electron is assumed to move independently in the nuclear Coulomb field and the average field of the remaining electrons.
- The electron-electron interaction is gathered by an appropriately chosen central potential $U(\mathbf{x}_{ij})$, where $\mathbf{x}_{ij} = |\mathbf{x}_i - \mathbf{x}_j|$.

Relativistic HF-Hamiltonian

$$H(\mathbf{x}_1, \ldots, \mathbf{x}_{N_e}) = \sum_{i=1}^{N_e} c\boldsymbol{\alpha} \cdot \mathbf{p}_i + \boldsymbol{\beta} c^2 - \sum_{i=1}^{N_e} \frac{Ze}{|\mathbf{x}_i - \mathbf{x}_N|} + U\left(\mathbf{x}_{ij}\right), \quad (11)$$

$$\text{with} \quad \boldsymbol{\alpha} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix} \quad \boldsymbol{\beta} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (12)$$

Wave function of the electron cloud

The N-electron wave function is defined as an antisymmetric product of one-electron Dirac orbitals $\phi_o(\mathbf{x})$ and is obtained by a Slater determinant:

$$\phi_{o_1 \dots o_{N_e}}(\mathbf{x}) = \frac{1}{\sqrt{N_e!}} \begin{vmatrix} \phi_{o_1}(\mathbf{x}_1) & \phi_{o_2}(\mathbf{x}_1) & \dots & \phi_{o_{N_e}}(\mathbf{x}_1) \\ \phi_{o_1}(\mathbf{x}_2) & \phi_{o_2}(\mathbf{x}_2) & \dots & \phi_{o_{N_e}}(\mathbf{x}_2) \\ \vdots & & & \\ \phi_{o_1}(\mathbf{x}_{N_e}) & \phi_{o_2}(\mathbf{x}_{N_e}) & \dots & \phi_{o_{N_e}}(\mathbf{x}_{N_e}) \end{vmatrix}. \qquad (13)$$

Every orbital $\{o_1 \dots o_{N_e}\}$ is specified by a set of three quantum numbers $\{\kappa_i, m_i, \mu_a\}$.

One electron orbitals

The dirac orbitals $\phi_o(\mathbf{x})$ are defined through spherical spinors $\Omega_{\kappa m}$

$$\phi_o(\mathbf{x}) = \frac{1}{r} \begin{pmatrix} P_E(r)\Omega_{\kappa,m}(\theta,\varphi) \\ iQ_E(r)\Omega_{\kappa,m}(\theta,\varphi) \end{pmatrix} \qquad (14)$$

Transition amplitude

Electron cloud transition

The electron transition amplitude according to **Migdal's approach** is

$$Z_{FI}(\mathbf{q_e}) = \sum_{\sigma \in S_{N_e}} \text{sgn}(\sigma) \sum_{\alpha_i=1}^{4} \prod_{i=1}^{N_e} \int d^3x_i \phi^{\alpha_i *}_{o^F_{\sigma(i)}}(\mathbf{x}) e^{-i\mathbf{q}_e \cdot \mathbf{x}_i} \phi^{\alpha_i}_{o^I_{\sigma(i)}}(\mathbf{x}), \quad (15)$$

where $\phi^{\alpha_i *}_{o^F_{\sigma(i)}}$ is either an excited or an ionized electron state.

Probability conservation

$$\sum_F |Z_{FI}|^2 = |Z_{II}|^2 + \underbrace{\sum_{n,l,n',l'} p_{q_e}(nl \to n'l')}_{\text{excitation}} + \underbrace{\sum_{n,l} \int \frac{dE_e}{2\pi} \frac{d}{dE_e} p_{q_e}(nl \to n'l')}_{\text{ionization}} \quad (16)$$

Transition amplitude at leading order

- The atomic recoil energy is smaller than $\mathcal{O}(100keV)$, such that the factor $|\mathbf{q}_e \cdot \mathbf{x}_i|$ is expected to be smaller than $\mathcal{O}(1)$ on the atomic scale.
- At leading order only one electron can be ionized/excited and only excitations with $\Delta l = |l' - l| = 1$ are allowed.

Leading order transition amplitude

$$
\begin{aligned}
Z_{FI}(\mathbf{q_e}) \approx & -i \sum_{\alpha_i=1}^{4} \int d^3x_i \phi^{\alpha_i *}_{o_{k'}}(\mathbf{x})(\mathbf{q}_e \cdot \mathbf{x}_i)\phi^{\alpha_i}_{o_k{}^I}(\mathbf{x}) \\
= & -i\mathbf{q}_e \int dr\, r \times \left[P_{E_{i'}}(r)P_{E_i}(r) + Q_{E_{i'}}(r)Q_{E_i}(r)\right] \\
& \times \int d\Omega \Omega^{\dagger}_{\kappa' m'}(\theta,\varphi) \cos\theta \Omega_{\kappa m}(\theta,\varphi)
\end{aligned}
\tag{17}
$$

Isolated nuclear recoil

Yukawa contact interaction

The simplest effective Lagrangian to estimate the atomic recoil is a spin independent (SI) Yukawa interaction Lagrangian

$$\mathcal{L} = \sum_{i=p,n} \frac{g_i}{M_*^2} \bar{\psi}_i \psi_i \bar{\psi}_{DM} \psi_{DM}, \tag{18}$$

where M_* is a mass parameter and $g_{p,n}$ are dimensionless coupling constants.

Differential recoil cross section

$$\frac{d\sigma}{dE_R} = \frac{1}{32\pi} \frac{m_A}{\mu_N^2 v_{DM}^2} \frac{|F(\mathbf{p}_A)|^2 |\mathcal{M}(\mathbf{p}_A)|^2 |Z_{FI}(\mathbf{q_e})|^2}{(p_A^0 + p_{DM}^0)^2} \tag{19}$$

The Migdal effect in the above analysis is formulated, such that the atomic recoil cross section is obtained coherently

Kinematical constraints

For ionization and excitation, the process is no longer elastic:

$$E_{ec}^F \neq E_{ec}^I \tag{20}$$

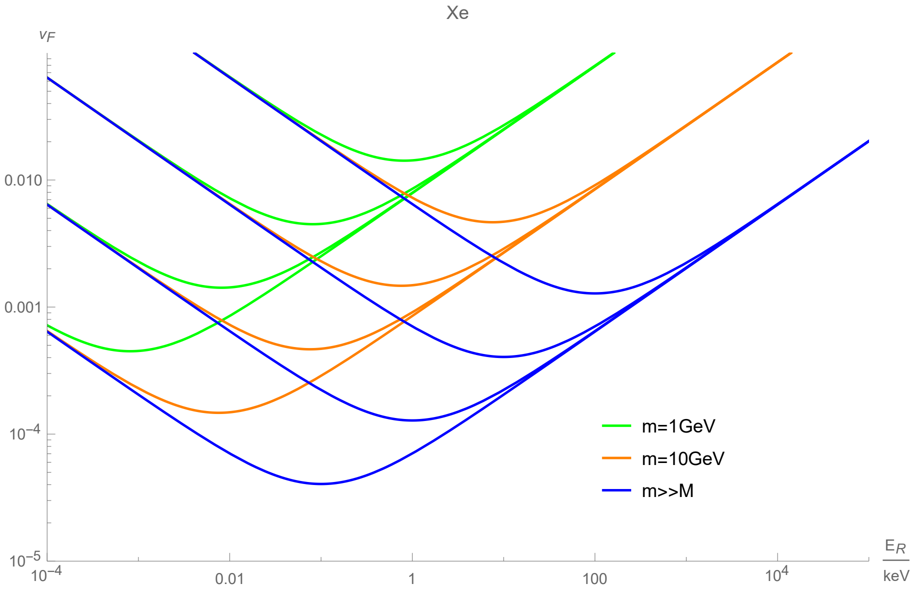
Kinematical constraints

$$E_R = \frac{\mu_N v_{DM}^2}{2m_N}\left(\left(1-\sqrt{1-\frac{2\Delta E}{\mu_N v_{DM}^2}}\right)^2 + 2(1-\cos\theta)\sqrt{1-\frac{2\Delta E}{\mu_N v_{DM}^2}}\right),$$
$$\Delta E = E_e + E_{nl} \tag{21}$$

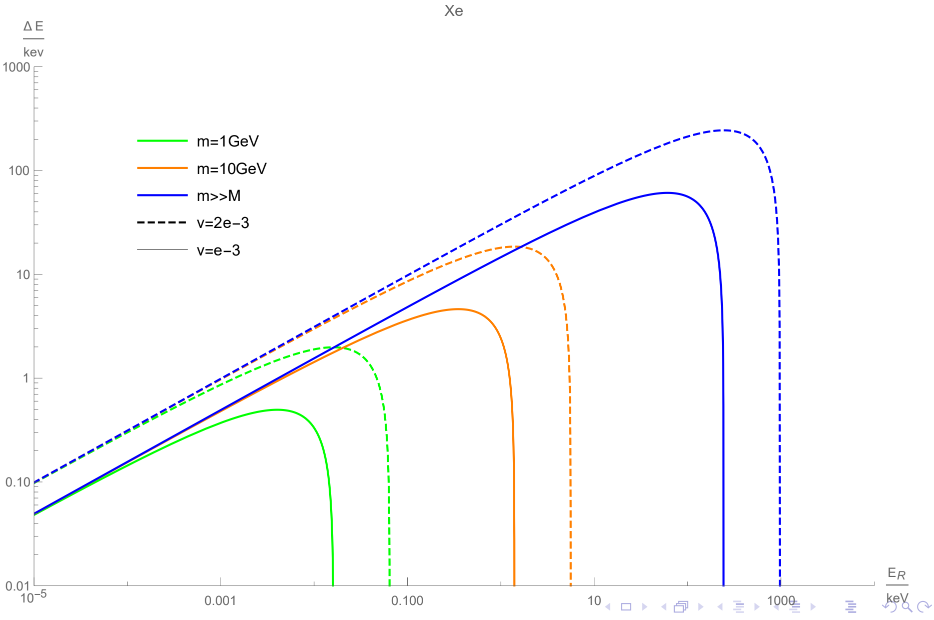
where E_e is the energy of the ionized electron and E_{nl} is the energy for the bound state electron. The DM-velocities are constrained by

$$v_{DM}^{(th)} = \sqrt{\frac{2\Delta E}{\mu_N}}, \qquad v_{DM}^{(min)} = \frac{m_N E_R + \mu_N \Delta E}{\mu_N \sqrt{2 m_N E_R}}. \tag{22}$$

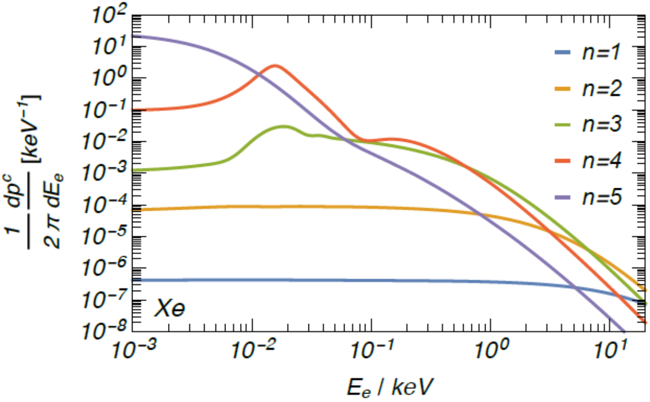
Kinematical Constraints -Xe



Kinematical Constraints -Xe



Ionization probability - Xe



$$\frac{d}{dE_e} p^c_{q_e}(nl \to E_e) = \frac{\omega_{nl}}{2(2l+1)} |Z_{FI}(q_e)|^2 \tag{23}$$

Dark Matter event rate

Dark Matter event rate per unit detector mass

$$\frac{dR}{dE_R dv_{DM}} \simeq \sum_{E_{ec}^F} \frac{1}{2} \frac{1}{\mu_N^2} \sigma_N(q_A) |Z_{FI}(q_e)|^2 \frac{\tilde{f}(v_{DM})}{v_{DM}}, \tag{24}$$

where $\tilde{f}(v_{DM})$ is the dark matter velocity distribution integrated over the directional component.

Ionization spectrum at leading order

$$\frac{dR}{dE_R dE_e dv_{DM}} \simeq \frac{dR_0}{dE_R dv_{DM}} \frac{1}{2\pi} \sum_{n,l} \frac{d}{dE_e} p_{q_e}^c(nl \to E_e) \tag{25}$$

$$\frac{dR_0}{dE_R dv_{DM}} \simeq \frac{1}{2} \frac{\rho_{DM}}{m_{DM}} \frac{1}{\mu_N^2} \sigma(q_A) \frac{\tilde{f}(v_{DM})}{v_{DM}} \tag{26}$$

References

[1] Landau, Lifshitz; *Quantum mechanics*; Third Edition; Pergamon Press (1976)

[2] Walter R. Johnson; *Atomic structure theory*; Third Edition; Springer (2006)

[3] arXiv:1707.07258 [hep-ph]

[4] arXiv:0706.1421 [hep-ph]